Precise Active Frequency Stabilization of a Microwave Cavity

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***Abstract*— In this paper, a system for precise frequency stabilization of a microwave cavity is presented. A cavity mode is probed in transmission, and a phase comparison scheme is used to measure its frequency fluctuations. In this way, shifts are corrected not only if temperature induced, but also if induced by mechanical stress, inasmuch as the latter effect is similar for the target mode and for the mode used to implement the stabilization system. A servo loop is then realized, which locks the cavity resonance to an external reference set to the nominal value by thermal actuators driven by a proportional-integral compensator.**

I. Introduction

In metrological experiments involving microwave cavities (i.e. atomic frequency standards, sapphire resonators, Fabry-Pérot cavities) targeted levels of accuracy and long term stability often require stabilization of the cavity resonance frequency. The latter depends mainly on the geometrical dimensions, which are subject to variations by thermal expansion or strain under mechanical stress. The standard approach consists in measuring temperature fluctuations or mechanical strains by dedicated sensors like thermocouple arrays, and strain gauges, and then closing the servo loop by suitable actuators ([1]). The method proposed here is based on a direct measurement of the displacement of the cavity resonance frequency from the nominal value, in order to obtain a signal proportional to the deviations form the nominal size of the cavity. A heating system, driven by a proportional-integral (PI) compensator, locks the cavity resonance to the predicted value.

The advantages of this control technique are the direct sensing of the cavity frequency, avoiding systematic effects of temperature or strain sensors placed on the cavity outer surface, and the accuracy of the control set point, provided by an external frequency reference.

Although the method was tested using the Ramsey cavity of a frequency standard based on the cesium fountain scheme, it is suitable for different stabilization problems.

II. Proposed method

The basic layout of the cavity resonance frequency measurement is shown in Fig. 1. The modal resonance frequencies ν_{0mnk} depend, among other things, on temperature and mechanical stress imposed to the cavity, i.e. $\nu_{0mnk} = \nu_{0mnk}(T,P)$. The phase shift introduced by a cavity excited at a frequency close to its modal resonances is equal to

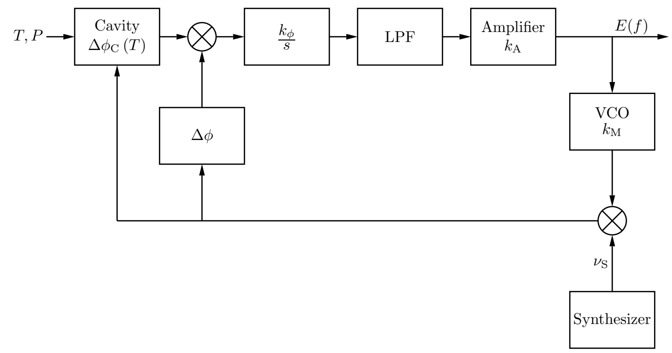

Fig. 1. PLL for cavity resonance measurement.

$$\Delta\phi_{\mathrm{C}} = k_{\mathrm{C}}\left(\nu_{0mnk}\left(T,P\right) - \nu_{\mathrm{i}}\right) \quad (1)$$

where $k_{\mathrm{C}} = 2Q/\nu_{0mnk}$ is the cavity phase sensitivity, Q is the resonant mode Q-factor, and ν_{i} is the input microwave frequency.

In order to obtain a signal related to the dimensional fluctuations of the cavity, a classical phase comparison scheme can be used.

A voltage controlled oscillator (VCO) signal is mixed with a microwave frequency source in order to obtain the cavity excitation signal, which feeds the cavity and a phase shifter. The output signals are mixed and then sent, via a low pass filter (LPF) and an amplifier, to the VCO, in order to realize a phase lock loop (PLL).

According to (1) the signal $E(f)$ in the scheme in Fig. 1 is

$$E(f) = \frac{k_{\mathrm{A}}k_\phi\left[k_{\mathrm{C}}\left(\nu_{0mnk}\left(T(f),P(f)\right) - \nu_{\mathrm{S}}\right) - \Delta\phi\right]}{1 + k_{\mathrm{A}}k_\phi k_{\mathrm{C}}k_{\mathrm{M}}}, \quad (2)$$

where k_{A} is the amplifier gain, k_ϕ is the mixer gain, k_{M} is the VCO modulation sensitivity, ν_{S} is the synthesized frequency, $\Delta\phi$ is a constant phase shift, and f is the Fourier frequency. Since the loop gain $k_{\mathrm{A}}k_\phi k_{\mathrm{C}}k_{\mathrm{M}}$ is greater than 1, eq. (2) yields

$$E(f) = \frac{\nu_{0mnk}\left(T(f),P(f)\right) - \nu_{\mathrm{S}}}{k_{\mathrm{M}}} - \frac{\Delta\phi}{k_{\mathrm{C}}k_{\mathrm{M}}}. \quad (3)$$

The signal $E(f)$ is sent, through a PI compensator, to the heating system which modifies, by thermal expansion, the cavity size and sets its modal frequency to ν_{S}.

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TABLE I
CAVITY MODES DATA.

Mode	ν_{0mnk}/ GHz	Q	IL /dB	Notes
TE 211	8.4889	10^4	70	
TE 011	9.1926	2×10^4	58	clock mode
TE 121	10.8399	2×10^4	90	

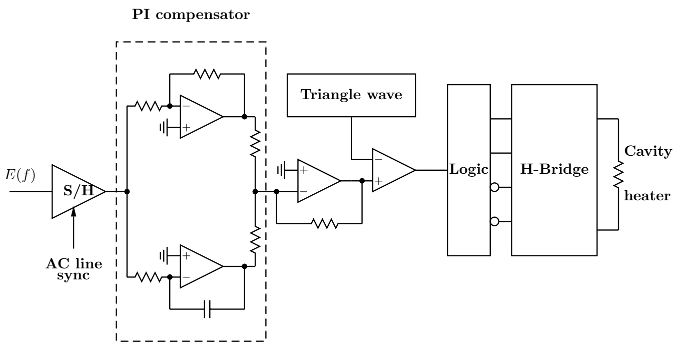

Fig. 2. Compensator and actuators driver for cavity frequency servo loop.

III. EXPERIMENTAL SETUP

The method described has been tested on the cavity of an atomic clock based on fountain scheme. Resonance modes of interest are listed in Tab. I. Both the TE_{211} and the TE_{121} modes are suitable for the frequency control. The choice of the TE_{211} mode at $8.4889\,\mathrm{GHz}$ is dictated by the smaller insertion loss (IL).

A $10\,\mathrm{MHz}$ VCO is used with a $8.4789\,\mathrm{GHz}$ microwave source to generate the probing signal. Calculated and measured sensitivities of the cavity with respect to temperature and to an external compression weight are, respectively, $150\,\mathrm{kHz/K}$ (for all modes) and $1\,\mathrm{kHz/kg}$ for the TE_{211} mode.

Because of the difference between the sensing sensitivity (proper of each mode) and the actuating sensitivity (the same for all modes), a detuning frequency offset for the clock mode may appear if mechanical stress acts on the cavity; for the considered modes, the sensitivity discrepancy is about 10%.

Fig. 2 shows the pole-dominant compensator, which realizes the desired transfer function, and the actuator stage. The heating system consist of two amagnetic heaters directly wound around the outer surface of the cavity. Since, for this particular application, no static magnetic field is admitted, the heaters are driven by a symmetrical pulse width modulation (PWM) waveform. A frequency of $20\,\mathrm{kHz}$ was chosen for the latter in order to make its period sufficiently smaller then the $7\,\mathrm{ms}$ flight time through the cavity. In this way, the average net DC field experienced by the atoms is adequately reduced. ([2]).

An additional sample and hold (S/H) stage, synchronous to the AC power supply, is inserted, in order to realize a notch-type filter for the AC line coupled interference.

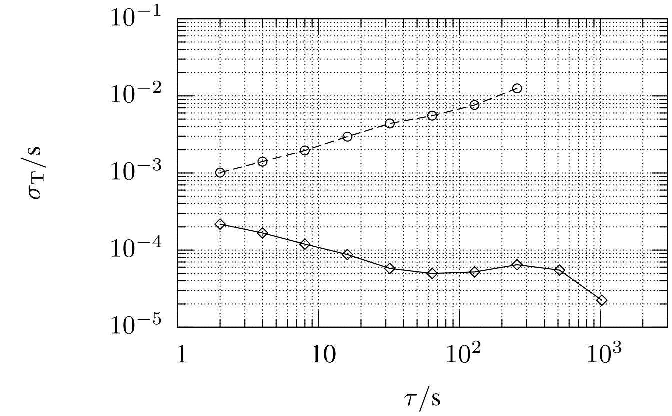

Fig. 3. Temperature deviation of the cavity versus measuring time τ: uncontrolled (upper curve), controlled (lower curve).

IV. RESULTS

A preliminary test of the proposed method concerned temperature stabilization of the cavity when no external thermal excitation is imposed. Fig. 3 shows the Allan deviation of the cavity temperature versus measuring time τ; the open loop operation shows a random-walk like process, while the servo loop behavior is limited by a $50\,\mu\mathrm{K}$ flicker floor (corresponding to 10^{-9} relative frequency shift), probably due to the temperature instability of the two arms of the phase comparison scheme.

Since the method was devised to control the temperature of the Ramsey cavity of a cesium fountain during an experiment aimed to the evaluation of black-body radiation shift ([3], [4]), the servo loop has to fulfil some requirements; in particular, when a $100\,\mathrm{s}$, $40\,\mathrm{K}$ peak-to-peak cycling temperature is applied on the drift tube of the fountain, in order to force a cyclical black-body frequency shift to the flying atoms, temperature fluctuation of $120\,\mathrm{mK}$ peak-to-peak is measured at the top of the cavity (Fig. 4 upper curve). The end-to-end cavity phase shift, caused by its temperature fluctuations, is then detrimental for the accuracy of the measured effect. A $50\,\mathrm{dB}$ rejection is necessary to reach the desired level of temperature stability. Under this requirement, the servo loop performs a $50\,\mathrm{dB}$ at $10^{-2}\,\mathrm{Hz}$ and a residual fluctuation of $270\,\mu\mathrm{K}$ peak-to-peak is measured (Fig. 4 lower curve).

The influence of mechanical stress imposed to the cavity was also considered. In particular, the effect of weight difference between cold and hot water, used to cycle the temperature of the drift tube, was measured, showing a negligible effect on clock mode frequency accuracy.

V. CONCLUSION

An active frequency stabilization of a cavity is presented. The method, tested for a microwave cavity, is suitable for different problems involving multi-modal resonators. The tested version shows a temperature equivalent stability limit of $50\,\mu\mathrm{K}$, corresponding to 10^{-9} relative frequency shift.

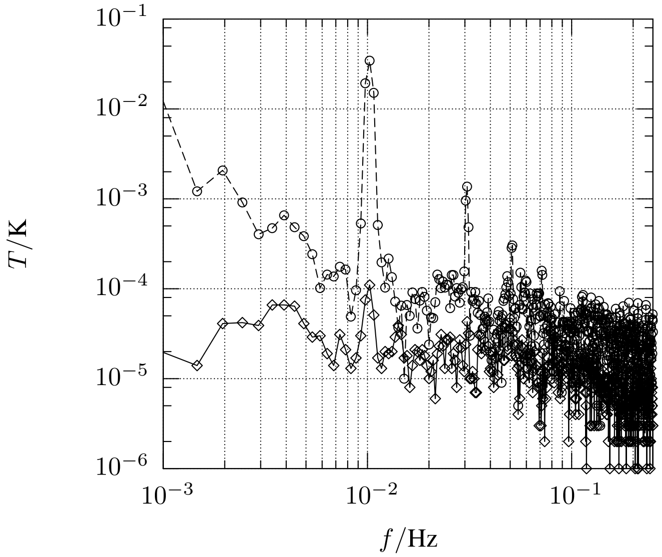

Fig. 4. Cavity temperature PSD with thermal cycles of $\Delta T = 40$ K peak-to-peak, 100 s period on the drift tube: uncontrolled (upper curve), controlled (lower curve).

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